Using Slash Modes for Cadences

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Authentic cadences are produced with a dominant seventh interpretation of a 7-chord and a cadence-raised target chord:

(1)
$$
\begin{array}{lll}
I & V^7 & I \\
\mathsf{I} : \mathbf{I} & \mathsf{V}^7/\mathsf{I}^7 : \lambda x.\mathit{leftonto}(x) & \mathsf{I} : \mathbf{I} \\
 & & \overline{(\mathsf{I}\backslash\mathsf{I})\backslash(\mathsf{Y}^7/\mathsf{I}^7) : \lambda C.\lambda P.P + C(\mathbf{I})}^{>\mathbf{T}} \\
 & \multicolumn{2}{c}{\overline{\mathsf{I}\backslash\mathsf{I} : \lambda x.x + \mathit{leftonto}(\mathbf{I})}^{<}} \\
\multicolumn{3}{c}{\overline{\mathsf{I} : \mathbf{I} + \mathit{leftonto}(\mathbf{I})}^{<}}
\end{array}
$$

Extended cadences can be interpreted by composing the dominant seventh categories:

(2)
$$
\begin{array}{llll}
I & II^7 & V^7 & I \\
\mathsf{I} : \mathbf{I} & \mathsf{II}^7/\mathsf{V}^7 : \lambda x.\mathit{leftonto}(x) & \mathsf{V}^7/\mathsf{I}^7 : \lambda x.\mathit{leftonto}(x) & \mathsf{I} : \mathbf{I} \\
 & \multicolumn{2}{c}{\overline{\mathsf{II}^7/\mathsf{I}^7 : \lambda x.\mathit{leftonto}(\mathit{leftonto}(x))}^{>\mathbf{B}}} & \overline{(\mathsf{I}\backslash\mathsf{I})\backslash(\mathsf{Y}^7/\mathsf{I}^7) : \lambda C.\lambda P.P + C(\mathbf{I})}^{>\mathbf{T}} \\
 & \multicolumn{3}{c}{\overline{\mathsf{I}\backslash\mathsf{I} : \lambda x.x + \mathit{leftonto}(\mathit{leftonto}(\mathbf{I}))}^{<}} \\
\multicolumn{4}{c}{\overline{\mathsf{I} : \mathbf{I} + \mathit{leftonto}(\mathit{leftonto}(\mathbf{I}))}^{<}}
\end{array}
$$

The dominant seventh category I^7_X/IV^7_X correctly indicates that a chord X^7 raises the expectation of a chord IV_X to follow it.

This combines nicely with the cadence-raising rule to produce interpretations of cadences because symbols X^7 only enter the derivation through dominant seventh categories like this (including also the tritone substitution categories). In particular, there is no category $X^7 := I^7_X$ that would give rise to something like the following, meaning that the X^7 symbols only combine with other categories by composition, or with the cadence endings by application.

(3)
$$
\begin{array}{ll}
II^7 & V^7 \\
II^7/V^7 & V^7 \\
\multicolumn{2}{c}{\overline{II^7}^{>}}
\end{array}
$$

However, with right steps – plagal cadences – we are not so lucky as to have a clear signal, like the minor seventh, or a distinct category symbol, like the X^7 symbols. As a result, as well as good interpretations like:

(4)
$$
\begin{array}{lll}
I & IV & I \\
I & IV/I & I \\
 & & \overline{(I\backslash I)\backslash(Y/I)}^{>\mathbf{T}} \\
 & \multicolumn{2}{c}{\overline{I\backslash I}^{<}} \\
\multicolumn{3}{c}{\overline{\mathsf{I} : I + \mathit{rightonto}(\mathbf{I})}^{<}}
\end{array}
$$

we get bad ones like:

(5)
$$
\begin{array}{lll}
I & IV & I \\
I & IV/I & I\backslash I \\
 & \multicolumn{2}{c}{\overline{IV\backslash I}^{<\mathbf{B}_\times}} \\
\multicolumn{3}{c}{\overline{\mathsf{I} : \mathit{rightonto}(\mathbf{I})}^{<}}
\end{array}
$$

The key problem is that the category 5, which gives us the plagal steps, is not intended to be combined by function application with its resolution, but is only meant to be used to build a plagal sequence, which should be eventually combined with its cadence-raised target.

What is needed is a way to constrain the ways in which these categories may combine with others. These categories may be involved in composition with other cadential categories, but may not act as functors in function application or compose with non-cadential categories. This is almost the opposite of the * modality used in natural language CCG grammars.

I therefore propose a single modality symbol "c" denoting that the category is cadential and so may only combine as described above. Other categories (implicitly) have a non-cadential modality. I will mark this mode here as "ϕ", but this is the implicit mode where none is given.

This gives us a new category 5:

$$5.\ \ X(m) := I_X(m)/_c V_X(m)$$

Similar changes are made to other cadential categories.

The combinatory rules must be modified to be conditional on the modality of their inputs as follows[1]:

$(>)$	$X/_m Y \;\; Y \Rightarrow X$	if $m = \phi$
$(<)$	$X \;\; Y\backslash_m X \Rightarrow Y$	if $m = \phi$
$(> B)$	$X/_{m_1} Y \;\; Y/_{m_2} Z \Rightarrow X/_{m_3} Z$	$m_3 = c$ if $m_1 = c$ or $m_2 = c$, o/w $m_3 = \phi$
$(< B)$	$X\backslash_{m_1} Y \;\; Z\backslash_{m_2} X \Rightarrow Z\backslash_{m_3} Y$	$m_3 = c$ if $m_1 = c$ or $m_2 = c$, o/w $m_3 = \phi$
$(> Bx)$	$X/_{m_1} Y \;\; Y\backslash_{m_2} Z \Rightarrow X\backslash m_1 Z$	if $m_2 = \phi$
$(< Bx)$	$X/_{m_1} Y \;\; Z\backslash_{m_2} X \Rightarrow Z/_{m_1} Y$	if $m_2 = \phi$
(Ta)	$X \Rightarrow (I_X\backslash I_X)\backslash(Y^7/_c I^7)$	
(Tp)	$X \Rightarrow (I_X\backslash I_X)\backslash(Y/_c I)$	

The result of this is to cut down the number of results produced by parses of cadences quite dramatically, since all of the poor uses of the plagal cadence categories are eliminated.

[1] It is not really necessary to use the cadential mode for authentic cadences, since they are already distinguished by the ^{7}s, but for now I am treating them in this way to be consistent with plagal cadences.